Geometric representation theory and low-dimensional topology – summer school

3 – 7 June 2019

International Centre for Mathematical Sciences, Edinburgh

Abstracts

Cooke, Juliet
Skein categories
In this talk we will introduce a categorical analogue of skein algebras based on coloured ribbon tangles. We shall then see how these skein categories satisfy excision and can be treated as k-linear factorisation homology theories of surfaces.

Gonzalez, Nicolle
The higher structure of the Heisenberg and Clifford algebras
An important role in the structure of CFT is played by vertex operators. Categorifying these operators is, thus, a natural step towards understanding the higher structure of CFT. As a motivating example, I will explain a possible approach via the categorification of the vertex operators which relate the actions of the Heisenberg and Clifford algebras on Fock space, known as the boson-fermion correspondence. In the process, I will briefly discuss Khovanov's Heisenberg category and some of the basics of diagrammatic categorification.

Ho, Quoc
Arithmetic applications of factorization homology
In recent years, the theory of factorization homology has emerged as a powerful tool to study problems in diverse areas of mathematics, including higher category theory, representation theory, manifold topology, and number theory (over function fields), etc. The last two topics will be the main focus of the talk. More specifically, we will explain how factorization homology can help clarify various links between homological stability phenomena and problems in arithmetic statistics. A review of the basics of factorization homology will also be provided.

Kanstrup, Tina
Knot homologies and matrix factorizations
One of the most famous link invariants is Khovanov-Rozansky triply graded homology. Its definition is completely algebraic and notoriously hard to compute. It has been conjectured by Gorsky, Negut and Rasmussen that it can also be computed as cohomology of certain coherent sheaves on the flag Hilbert scheme. A link invariant of a similar nature has been constructed by Oblomkov and Rozansky in terms of matrix factorizations. In this lecture series we will describe these different approaches and reformulate the work of Oblomkov and Rozansky into the setting of the conjecture using work of Arkhipov and Kanstrup. Time permitting we will relate this to the work of Ben-Zvi and Nadler et al. in derived algebraic geometry.

Masbaum, Gregor

Introduction to Skein theory

Skein modules were originally introduced to generalize the celebrated Jones polynomial of classical knots to links in other three-manifolds. They also appear naturally as quantizations of character varieties. The aim of this talk is to give an introduction to skein theory for non-experts and hopefully discuss some interesting open questions as well.

Minets, Alexandre

Cohomological Hall algebras and sheaves on surfaces

Cohomological Hall algebras (COHAs) where introduced by Kontsevich-Soibelman and Schiffmann-Vasserot about 10 years ago, and since then proved to be a useful language for unifying a number of constructions in geometric representation theory. I will start by explaining less technically challenging notion of Ringel-Hall algebra, and continue with the definition and examples of COHAs. If time permits, I will sketch some recent results about their action on cohomology of moduli of coherent sheaves on surfaces.

Safronov, Pavel

Shifted Poisson structures in representation theory

These talks are devoted to the theory of symplectic and Poisson structures on stacks developed by (Calaque—)Pantev—Toen—Vaquie—Vezzosi. In the first lecture I will introduce the language of derived algebraic geometry and define shifted symplectic structures. In the next lecture I will sketch a relationship with shifted Poisson structures and will spend the rest of the time on examples relevant for representation theory. These include: the Kirillov—Kostant—Souriau symplectic structure on coadjoint orbits, the Grothendieck—Springer resolution, Poisson—Lie structures on simple groups, reflection equation algebras, Sklyanin and Feigin—Odesskii algebras.

Schrader, Gus

The cluster approach to character varieties

It was shown by Fock and Goncharov that certain character varieties parametrizing framed SL(n)-local systems on surfaces bear the structure of cluster Poisson varieties. This observation allowed them to quantize these character varieties, and conjecture that this quantization satisfies the locality property of TQFT with respect to cutting and gluing of surfaces. I will discuss their construction, and explain the idea of the proof of their conjecture obtained in joint work with A. Shapiro.

Shapiro, Alexander

Around character varieties in 50 minutes

There are four main approaches to character varieties and their quantization: via skein algebras, via Alekseev – Grosse – Schomerus quantization of Fock – Rosly Poisson structure, via cluster algebras due to Fock and Goncharov, and via factorization homology due to Ben-Zvi – Brochier – Jordan. I will discuss how the latter three approaches are related to each other. The talk will be based on a joint work in progress with D. Jordan, I. Le, and G. Schrader.

Simental Rodriguez, Jose

Rational double affine Hecke algebras

Rational Cherednik algebras, or rational DAHA, are a degeneration of DAHA introduced by Etingof and Ginzburg around the year 2000. We will explore the precise connection between RCA and DAHA, and review some of the basic structural and representation theoretical properties of the RCA. Time permitting, we will also see the connection between the type A RCA and the Hilbert scheme of points in the plane.

Tanaka, Hiro Lee

Factorization homology, infinity-categories, and topological field theories

These lectures will serve as an introduction to the topics in the title. Factorization homology is a method for producing field theories out of rich algebraic gadgets; for example, it simultaneously yields invariants of manifolds and of higher algebras. We will also give peeks into the language of infinity-categories, which are necessary in one guise or another to articulate the rich spaces that cohere operations. Topological field theories will also be presented, but from a mathematics-centred (rather than physics-centred) perspective. The (ambitious) goal is to give examples that illustrate how these notions help to organize phenomena in representation theory.

Tolmachov, Kostiantyn

Knot homologies and geometric Hecke categories

I will describe geometric categorifications of the Hecke algebras coming from representation theory, and survey how the knot homologies are visible from the point of view of these categorifications, following the work of Webster and Williamson. Time permitting, I will also talk about the recent progress in relating the coherent categorification of the affine Hecke algebra with the constructible categorification of the finite Hecke algebra in type A, joint with Bezrukavnikov.

Vazirani, Monica

Hecke algebras and representation theory

The double affine Hecke algebra (DAHA) was invented by Cherednik in order to study (symmetric) Macdonald polynomials. This gave rise to the nonsymmetric Macdonald polynomials which gives a basis of (Laurent) polynomials in X's. The X's form a "third" of the DAHA in its triangular decomposition. They are also a weight basis for polynomials in Y's, which comprise another third. The middle third is the finite Hecke algebra, which in type A deforms the symmetric group Sn. The X's also form an irreducible and faithful representation of the DAHA for generic parameters q; t. One can generalize from here to studying other irreducible representations (irreps) that admit a Y -weight basis - so-called Y-semisimple or calibrated representations. Or one can study irreps on which the Y's act locally finitely - analogue of Category O. How can we understand, study, classify, or construct such irreps? This 4-hour course will focus on type A. We will start with some of the more classical representation theory and combinatorics for the symmetric group Sn, the (extended) affine symmetric group, Hecke algebras as their q-deformations, and possibly other ways Hecke algebras "arise in nature." We will examine the structure of affine Hecke algebra (AHA) representations, particularly using the functors of induction and restriction. Then we shall apply these tools toward DAHA representations. As time and interest permits, topics that may be covered include: Y –semisimple irreps, finite dimensional irreps, analogues of Schur-Weyl duality, monomial expansion of Macdonald polynomials, the spherical DAHA (possibly with connections to the elliptic Hall algebra or skein modules), double a affine braid group, orthogonal polynomials, action of SL2(Z) on DAHA.